METHOD OF AMPLITUDE-PHASE CHARACTERISTICS FOR ANALYZING CLIMATE DYNAMICS

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For diagnosis of the Earth's climate system a special method of amplitude-phase analysis of its dynamics is proposed. Based on the proposed method we identified the features of the structure of the zonal temperature field of the system consisting of the atmosphere and the underlying surface as well as its dynamics according to empirical data in the annual course and in the long-term variability for the Northern Hemisphere.

PROBLEM FORMULATION

Development of global monitoring of the Earth's climate system (ECS) gives rise to the need for the development of direct methods of diagnosing its global dynamics based on empirical data. Besides analysis of the functional data on the ECS, for its investigation (diagnosis and forecast) we need empirical and theoretical analysis of the climate derivatives which characterize the space-time structure of the system. This article proposes a method of amplitude-phase characteristics for analyzing the dynamics of the climate system. The method is based on determining the dynamic parameters of the ECS and analysis of their space-time distribution. Essential in the ECS is the presence of cycles of external conditions and internal phenomena. Therefore, along with amplitude analysis of its dynamics we propose the use of phase analysis. The purpose of the method is to determine the structural features in the synergetic dynamics of the system and to identify the global mechanisms of the formation of ECS with a quantitative estimate of their relative role.

The proposed method is used here in a spatially two-dimensional (latitude and altitude) variant for determining the thermodynamics of a zonally average system consisting of the atmosphere and underlying surface based on mean monthly data at different levels in the atmosphere in the yearly course and in the long-term variability. For the analysis we used temperature data of [3] (1958-1963) and [4] (1963-1973) for the Northern Hemisphere.

DEFINING THE METHOD OF AMPLITUDE-PHASE CHARACTERISTICS FOR ANALYSIS OF THE DYNAMICS OF THE GLOBAL CLIMATE

The ECS is a complex inhomogeneous system with a fairly wide range of variations of variables and parameters in space, which makes its analysis difficult. This work proposes a special method of amplitude-phase analysis for determining the dynamics of the ECS. Variations of the climate variables should be viewed against the background of the mean or any actual inhomogeneous distribution.

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this case the local background (mean annual regime or regime of some selected state of the system, relative to which we view its variations), which vary from point to point; it is excluded and does not interfere with analysis of the time variations of climate variables.

The dynamics of the ECS can be characterized by the following: 1) a phase parameter, for example, the time of achieving the variable \( x \) at the corresponding latitudes \((\phi)\) and altitudes \((z)\), 2) an amplitude characteristic, the advance of the front of variation of the variable \( x \) by a certain value \( \Delta x \), for example, in the yearly course. Additional or alternative variants of the set of dynamic characteristics are possible. Of interest, for example, is the analysis of the dynamics of the phases of extremes or extreme derivatives.

In this work the dynamics of the ECS are studied concretely with the aid of the amplitude-phase method on the example of the dynamics of the temperature \((x = T)\) regime (thermodynamics) of the system consisting of the atmosphere and underlying surface in the yearly course. Analyzed in particular are the dynamics of fronts of atmospheric heating by \( \Delta T = 1; 5K \) relative to January and cool fronts by \( \Delta T = -1; -5K \) relative to July. The amplitude characteristics of the dynamics of heating and cooling of the system in the annual course can be determined relative to any month, relative to the mean annual regime, or relative to extreme regimes. Variation of the value of \( \Delta T \) is necessary to explain nonlinear effects of the influence of the inhomogeneity of the ECS.

The proposed algorithms of programs of phase and amplitude analysis of the dynamics of the system are fairly simple. We will illustrate them based on analysis of heating (similarly for cooling) of the ECS in the annual course according to zonal mean monthly data for temperature at different levels in the atmosphere.

Based on data in [3] we determined the values of the temperature heating relative to January in all subsequent months at different isobaric surfaces (to 50 mbar) in different latitude zones of the Northern Hemisphere. Then, in the two-dimensional latitude-altitude space we drew isochrons of heating by a certain value \( \Delta T \) (amplitude analysis) in the corresponding month relative to January. The data used are spaced by latitude \((\Delta \phi = 5')\) and altitude \((\Delta z\) from 0.1 to 4 km). Therefore in drawing isochrons we used the method of spatial linear interpolation. The altitude of the corresponding isobaric surfaces in the annual course was determined on the basis of data in [3] about geopotential. The location of the isochrons, heating fronts, at subsequent moments of time (months) characterizes the thermodynamics of the ECS. In this case in the plane of spatial variables (latitude-altitude) time plays the role of a parameter.

In view of the spacing of the data in time \((\Delta t = 1\) month) spatial interpolation is also possible in describing the fine structure of the ECS. For identification (qualitative) of inhomogeneities in the system of the atmosphere and underlying surface with typical scales of \( \delta y \) with respect to space \((y\) is the spatial variable\) and \( \delta t \) with respect to time \((t)\) with typical rate of movement of isochrons \( u(y, t) \) the spacing of the utilized data in space \( \Delta y \) and time \( \Delta t \) should not exceed some limit:

\[
\Delta y \leq u \delta t, \tag{1}
\]

\[
\Delta t \leq \frac{k_v}{u}. \tag{2}
\]

For adequate (quantitative) description of the dynamic structure of the ECS strong inequalities must be fulfilled:

\[
\Delta y < u \delta t, \tag{1a}
\]

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Useful information about the structure of the system can be obtained from analysis of the integral phase characteristics of periodic, but generally speaking, anharmonic processes in the system, particularly with small nonlinear distortion factor, a measure of the nonsine nature of the periodic function. Phase analysis is suitable for investigating the dynamics of the ECS in the annual course with corresponding major harmonic in the equatorial areas. In the atmosphere of the Northern Hemisphere according to data of [3] from 1000 mbar to 50 mbar north of 20°N at least 90% of the temperature variability in the annual course is explained by the annual harmonic, except the levels higher than 200 mbar to the south of 50°N.

The structure of the ECS can be judged according to the degree of asynchronicity of the achievement, for example, by the temperature regime in its different zones of a definite phase of the annual course. In particular, this work analyzes the times of achievement in different zones of the system of the atmosphere and underlying surface of the 0-phase (moment of spring transition with $\frac{dT}{dt} > 0$ of the local temperature through the local mean annual regime) and the $\pi$-phase (moment of autumn transition with $\frac{dT}{dt} < 0$ of the local temperature through the local mean annual regime). Local moments of achievement of the corresponding phases were determined by linear interpolation with respect to time. Then with the aid of spatial linear interpolation in the two-dimensional latitude-altitude space we drew isochrons of the definite phase for subsequent moments of time (months in the annual course). The density of the isochrons characterizes the structure and thermal inertia of various zones of the system consisting of the atmosphere and underlying surface. Similarly we can analyze the time of achievement of extremes (1/3-phase and 2/3-phase) of temperature in the annual course, but with fairly large time spacing ($\Delta t = 1$ month) of the utilized data it is preferable to analyze the 0-phase and $\pi$-phase.
RESULTS

For analysis, besides the temperature data [3] in the atmosphere we also used surface data [2] in the annual course for the Northern Hemisphere.

Phase Analysis

Figure 1a shows the dynamics of isochrons of the spring (0) phase of the temperature regime of the atmosphere-surface system of the Northern Hemisphere in the annual course. The isochrons are marked by numbers which designate the time of achievement (in months) of the corresponding mean annual regime by the local temperature in the annual course.

It is evident from Fig. 1a that "spring" extends from the equator to the subtropics in the troposphere and simultaneously runs from the polar stratosphere. The latter can be related to ozone heating of the stratosphere. In polar latitudes the maximum of ozone content in the atmosphere is lower than in the middle and lower latitudes, and this position corresponds well to the identified feature of heating of the polar stratosphere (see for example, [1]). In the second half of April (4.2 months) in middle latitudes (about 55°N) heating of the atmosphere begins from the surface of the convective type. At the beginning of May (4.55 months) from the isochrons of the spring phase a general isochron is formed from the equator and from the surface of middle latitudes. In this case we isolate the subtropical area of high pressure within the boundary layer (1.5 km), for which "spring" sets in later. As a result of the convergence of the isochrons which move from the surface and from the stratosphere, "secondary" isochrons are formed: the spring phase extends into the polar stratosphere and upper troposphere of extratropical latitudes south of 50°N. In this case we identify a delay in the heating phase of the zone of the polar jet stream (about 65°N at an altitude of about 6-7 km, for example, see [1]). For comparison Fig. 1a gives the time scale of the achievement of (tₘ) by the insolation in the annual course at different latitudes of the corresponding mean annual values.

Similarly, Fig. 1b shows the dynamics of isochrons in the autumn (π) phase for the Northern Hemisphere according to temperature data [3] in the annual course. Comparison of Figs. 1a and b shows that the phase dynamics of temperature heating of the atmosphere-surface system and its cooling in the annual course differ fundamentally. If the spring phase is characterized to a significant degree by heating of the atmosphere of the convective type from the surface and heating of the stratosphere, the autumn thermodynamics have a substantially advective nature. The isochrons of the autumn phase have the form of vertical fronts, moving from the pole to the equator. In this case a counterfront of the π-phase moves from the equator above the boundary layer. As a result the polar front of the π-phase breaks in two, continuing to move toward the equator: above the boundary layer and within it. We note the appearance (9.8 months) of the polar jet stream at an altitude of about 7 km near 65°N [1]. We should also note that from latitude to latitude the phase fronts extend more rapidly above the boundary layer than within it.

We can estimate the typical velocities of advance of the 0- and π-phases for the temperature regime of the atmosphere of the Northern Hemisphere in the annual course: the horizontal velocity \( u_h \approx 10 \text{ km/hr} \) (70°N latitude/month), vertical velocity \( u_v \approx 10 \text{ m/hr} \). In this case the value of \( u_h \) is close to the typical velocity of advance of the 0- and π-phases for insolation \( u_s \approx 70° \text{ latitude/month} \).

The ratio of vertical and horizontal velocities \( \frac{u_v}{u_h} \approx 10^{-1} \) corresponds to the ratio of vertical (H=10 km) and horizontal (\( \frac{5 R}{4 R} \approx 10^4 \text{ km, R is the Earth's radius} \)) scales for the troposphere. With the found velocities for qualitative identification of the thermal features in the atmosphere with typical scales \( \Delta t \approx 1 \text{ month} \) according to (1) it is necessary to have a resolution of \( \Delta u_v \leq 7 \text{ vertically and } \Delta u_s \leq 70° \text{ monthly} \).
Fig. 2. Dynamics of isochronous heating of zonal atmosphere in annual course by 1K (a) and 5K (b) relative to the January temperature regime.

horizontally. For the data used [3] these conditions are fulfilled: $\Delta \varphi = 5^\circ \leq 70^\circ$, and $\max \Delta y = 4 \text{km} \leq 7 \text{km}$.

Amplitude Analysis

Figure 2a gives the amplitude analysis of the dynamics of the temperature régime, the heating of the atmosphere of the Northern Hemisphere in the annual course by $\Delta T = 1 \text{K}$ relative to the mean January régime. The dynamics of 1K warm fronts are determined by the mutual position of the isochrons at subsequent moments of time. In this case time plays the role of a parameter whose value for isochrons is presented in the figures in months. According to Fig. 2a, the heating of the atmosphere by $\Delta T = 1 \text{K}$ relative to the January régime extends from the surface of the highest latitudes (their sensitivity is greater) accessible to the Sun in February and subsequent months. The warm front extends into the middle troposphere and toward the equator, displaying the property of the subtropical area of high pressure*. The "collision" of the front which moves from the surface with the warm front from the stratosphere gives rise to two "secondary" warm fronts. One of them has an advective nature and rapidly extends toward the pole, revealing a subpolar property. The second warm front extends toward the equator.

A thermal property is revealed, associated with the break in the tropopause and with the subtropical jet stream (SJS) at altitudes of 12-14 km. Circles mark the altitudes of the tropical and polar tropopause in the mean annual régime at the corresponding latitudes [1]. In May (isochron 5) 1K heating relative to January practically surrounds the area associated with the SJS. After this there is a break in the warm front: closure of one of its areas around the thermally distinctive area and stabilization of the second area around the equatorial troposphere. We note that although in May the advance of the warm front toward the equator is different at different altitudes in the troposphere, the profile of the warm front then equalizes.

*Isochrons with intervals between them less than a month are drawn with dashed lines with the use of linear interpolation with respect to time.
Fig. 3. Dynamics of isochrons of cooling of zonal atmosphere in annual course by -1K (a) and -5K (b) relative to the July temperature regime.

of the temperature heating or cooling. For comparison with Fig. 2a, Fig. 2b gives the corresponding warm fronts at $\Delta T = 5K$ relative to the mean January regime. Certain features appear in comparing the dynamics of the temperature variations at different amplitudes with phase dynamics. In Fig. 2b the heating moves from the surface of mean and high latitudes and simultaneously from the polar stratosphere. After convergence of these two fronts the heating gives rise to "secondary" fronts: toward the pole (dot-dash line shows the warm front by $\Delta T = 3.5K$ in April relative to January) and the front moving toward the equator. By July the equatorial atmosphere is heated by 5K only at altitudes higher than 20 km. The atmospheric zone associated with the SJS and superjacent tropical tropopause is not subjected to 5K heating (relative to January) by July.

The-dashed line in Fig. 2b shows the boundaries of heating (and cooling) in July (7) relative to January at different $\Delta T$ from 4K to -3K (value of $\Delta T$ is shown in parentheses). With consideration of the altitude spacing (to 4 km) of the levels for which the data are presented in [3,4], the thermal property of atmospheric heating practically coincides with the tropical tropopause. (Altitudes of the mean annual tropopause, as in Fig. 2a, are marked with circles.) Besides the altitude zone associated with the SJS, this also appears in tropical latitudes: the heating structure differs by $\Delta T = 2K$ and $\Delta T = 3K$ in the vicinity of the tropical tropopause of altitudes of about 16 km [1].

Similarly, Figs. 3a and b analyze the dynamics of atmospheric cooling of the Northern Hemisphere in the annual course, respectively, by $\Delta T = -1K$ and $\Delta T = -5K$ relative to the mean July regime. Both in Fig. 2a and b, a thermal property is revealed which is associated with the break in the tropopause and SJS. As a whole the process of atmospheric cooling in autumn has an advective nature unlike spring heating (Figs. 2a and b), which has a significant convective nature. We note that the temperature warm and cool fronts, like phase fronts, extend more rapidly from latitude to latitude above the boundary layer than within it.

As evident in Fig. 3, the atmospheric cool front differs more from vertical with the increase in amplitude of $\Delta T$ in terms of absolute value. Deformation of the front increases as we approach the zone associated with the break in the tropopause and SJS. According to Fig. 3a, in October (10) there is a break in the cool front by $\Delta T = -1K$ relative to July: closure of one part of it around the thermally distinctive area and stabilization of the second part around the equatorial troposphere similarly to warm fronts in Fig. 2a. In this case the atmospheric zone
Fig. 4. Annual course of temperature according to data of [3, 4] in the zone of the subtropical jet stream (a) and in the middle troposphere (b) and corresponding long-term variability (c) and (d) and its spectrum (logarithm of spectral density $S_{TT}(f)$ [K$^2$·year] where $N = 15$) (e) and (f) according to data in [4] for July and January.

associated with the SJS and superjacent tropical tropopause is not subjected to 5K cooling relative to July. As in Fig. 2b, in Fig. 3b the structure of January isochrons of cooling by $\Delta T = -2$K and by $\Delta T = -3$K relative to July in the tropics is different in the vicinity of the tropical tropopause. A feature of Figs. 3a and b is the "reflection" of the cool front from the indicated zone associated with the SJS and the superjacent tropopause during November-January. Moreover, in the troposphere below 10 km the front of cooling by $\Delta T = -5$K advances successively toward the equator.

Variability Throughout the Year

The thermal property of the atmospheric zone associated with the SJS* is evident in Fig. 4, which presents the mean annual course of temperature according

*We note that at the level of the tropical tropopause the annual course of temperature is practically in counterphase to the annual course of temperature in the middle troposphere.
to data of [3] and [4] at the level of the SJS (a) and in the middle troposphere (b). The effect of the "reflection" of the cool front for the zone of the SJS appears with negative (winter) deviations of temperature relative to the mean annual regime. As a result, like systems of the trigger type, the regime of negative deviations relative to the mean annual regime in the SJS zone expands in time, and the regime of positive deviations is shorter and with a greater amplitude. For comparison, Fig. 4b presents the annual course of temperature in the middle troposphere (at 40°N at the level of 500-mbar) with dominant annual harmonic.

It would be interesting to establish how the thermal features of the atmosphere in the annual course affect the variability of the temperature regime. Fig. 4c and d present graphs of the long-term temperature variability in the zone of the SJS (c) at 40°N at the level of 200 mbar and in the middle troposphere (d) at the same latitude at the level of 500 mbar according to 15-year data [4] for July and January. We note first of all that the amplitude of the long-term temperature variability in the zone of the SJS (±2K) is twice that in the middle troposphere (±1K). If we consider that the spread of amplitudes of the annual course of temperature consists of about 6 and 16K, respectively, the ratios of the amplitude of long-term variability to the amplitude of the annual course differ by more than 5 times: 2/3 and 1/8. It is obvious that the long-term variability is substantially more significant in the zone bounded by the SJS. This primarily affects the long-term variability of temperature in January.

It is clear in Fig. 4c that the long-term variability of January and July temperatures in the SJS zone differs markedly. Besides the noted differences in amplitudes, we find a difference in the typical periods of long-term variability: about 10 years for January and 2-3 years for July temperatures. Similar differences are not observed for the middle troposphere (Fig. 4d). Thus, with the presence of the effects of "reflection" in the period of cooling of the atmospheric zone associated with the SJS, there seems to be an increase in the thermal inertial of this zone in winter compared to summer, which also appears in the long-term variability.

Figure 4e and f present the form of the spectral function $S_{\omega \nu}(\omega)$ for the long-term variability of temperature according to the data in Fig. 4c and d, respectively, at the level of the SJS (40°N, 200 mbar) and in the middle troposphere (40°N, 500 mbar). In this case we used* Berg's method of spectral evaluation with a fifth-order autoregression model. Identified in Fig. 4e at the level of the SJS, as indicated above, is the dominant period (τ) for both January (about 10 years) and July (2-6 years) temperatures. We should note that the maxima in January and July spectra in Fig. 4e are separated, and the spectra seem to complement each other. Based on the form of the spectra we can assume that in the discussed area at the level of the SJS in winter and summer different temperature regimes of long-term variability are realized. In this case the typical time scale of long-term dynamics of the summer regime (with greater atmospheric temperature of the Northern Hemisphere as a whole, a smaller equator-pole temperature gradient, and less intense general circulation in the atmosphere) is smaller than that of the winter regime.

We note that the 15-year series of data utilized is short and the obtained spectra may serve only for preliminary qualitative analysis. The sensitivity of the results of spectral calculations can be estimated by varying the order of the

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autoregression model. With its variation from 5 to 7 the maximum in the January spectra is shifted from 11 to 9-8 years, but in the July spectrum there is no displacement. Regimes with these periods remain clearly dominant in the spectrum, although local maxima with other periods do appear. Similar regimes with a dominant period are not obtained in the corresponding spectra in the middle troposphere (40°N, 500 mbar) in Fig. 4f.

CONCLUSIONS AND PROSPECTS

The results obtained indicate that the proposed method is a useful and sufficiently general means for investigating the ECS and its dynamics. This is shown here on the example of the zonal temperature field of the system consisting of the atmosphere and underlying surface. The features of the thermal structure of this system and its dynamics in the annual course and in the long-term variability are identified.

In particular, the structure of the dynamics of heating and cooling of the atmosphere in the annual course corresponds to the geometry of the polar and tropical tropopause. The thermal feature of the atmospheric zone associated with the SJS and the subjacent tropical tropopause is established by isochrons of heating relative to January (or cooling relative to July), which converge toward this zone from the stratosphere and troposphere. Depending on the conditions the variation of the temperature regime of the atmosphere-surface system in the annual course has an advective or convective character. In particular, the temperature dynamics differ in spring and autumn. We also identified the features of the boundary layer, equatorial troposphere, subtropical high-pressure zone, and polar jet stream.

We estimated the typical space-time scales of thermodynamics of various zones of the discussed system. The typical velocity of horizontal heating (or cooling) of the troposphere in the annual course is on the order of 10 km/hr, and that of vertical heating is on the order of 10 m/hr. Typical for the temperature regime of the trigger type. The difference in the dynamics of heating and cooling of this zone in the annual course corresponds to the difference of the long-term variability for January and July temperatures with respect to amplitude and typical periods of variation. Spectral analysis should be used along with the amplitude-phase method, and we should study the dynamics of individual modes of the system (not necessarily harmonic) with the investigation of free and induced fluctuations, phenomena of the parametric resonance and pulsation, and regimes of auto-fluctuations.

For more detailed analysis of the dynamics of the ECS we need more detailed empirical data of longer measurement series with smaller space-time scales. In particular, for the upper troposphere and lower stratosphere we need data with less distance between the levels vertically, otherwise to identify the temperature feature with the typical scale on the order of 4 km (distance between levels in the upper troposphere) we need data with time resolution no greater than half a month.

In this work we analyzed a spatially two-dimensional (latitude-altitude) system. We can similarly analyze other two-dimensional (latitude-longitude), (longitude-altitude) and three-dimensional (latitude-longitude-altitude) systems. In the last case the dynamics of the system are characterized by the mutual position of the successive isochron surfaces of heating (or cooling) and play the role of their determining parameter at that time.

The method of amplitude-phase analysis of dynamics is formalized, and with its aid we can study inhomogeneous and mobile fields of different variables, even
in geophysical thermodynamics and hydrodynamics. Like analysis of the dynamics of the temperature field in the atmosphere we can study the field of humidity and different atmospheric components (ozone, CO₂, etc.), velocity, pressure, temperature and salinity in the ocean, flows of heat, moisture, etc.

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